

A Quick and Dirty Approach to Verify the Pioneer Anomaly

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March 2, 2007

Abstract

We present source code for the computer algebra system Mathematica that analyzes the motion of the Pioneer spacecraft using the public available ephemeris data from JPL's website. Within 15 minutes, the reader can verify that the Pioneer anomalous acceleration a_p (1) exists in the order of magnitude of cH_0 , (2) is not due to mismodeling of gravitational attraction, solar pressure or spacecraft attitude maneuvers. The simple code of about 100 lines may easily be extended by the reader to include further tests. Due to the limitations of our approach, we do not know (1) whether the unknown raw data were correctly processed to generate the trajectory files (2) how the apparent mismatch of ephemerides before 1990 had occurred.

1 Introduction

The Pioneer anomaly $a_p \approx -8.7 \times 10^{-10} \text{ms}^{-2}$ [1, 2] has become one of the major challenges for theoretical physicists, and further efforts to investigate its origin are underway [3, 4, 5, 6]. The approximate coincidence of a_p with the speed of light divided by the age of the universe has raised the question if this effect is new physics or a general failure of Newton's law of gravitation [2, 7, 8, 9]. A couple of months ago, one of us heard two astrophysicists talking about the anomaly. One of them, probably tired of hearing about new trouble besides DM and DE, concluded: 'well, I still don't believe it...'.

Though the analysis of the Pioneer data was done by two independent groups and published in peer-reviewed journals, it is reality that convincing scientists needs time. Here we do not present any new results that help to understand the anomaly, and our approach cannot compete with the detailedness of the expert's analysis [2, 7] of the non-public raw data. From a point of view of general scientific methodology, we find it however desirable that important results of fundamental physics that require extensive numerical treatment can be repeated by a broad public of non-expert scientists. The preliminary analysis and the code given below is indeed intended for those who like to get their own opinion in brief. Furthermore, minor modifications allow to test some alternative explanations the reader eventually may have in mind. A quick description for getting started is found in section 6.1. Though we cannot give a detailed description of the program, some clarifying comments are included in the quite self-explaining code (see appendix).

2 Methods

2.1 Limitations

Our analysis is based on the reliability of JPL's ephemeris files. As far as Pioneer 10 and 11 is concerned, they contain an explicit warning:

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‘This trajectory is suitable for general historical purposes, but should be used cautiously for high precision or tracking data applications. This is due to potential dynamical mismatches between the Pioneer era models (DE-118, Lieske’s E3 satellite theory of JUP035, SAT050, etc.) and the current modern solutions used by Horizons. For example, if the Pioneer 10 (11) solutions used here estimated planet or satellite ephemeris corrections at the time. However, the transformation from the original DE-118 planetary ephemeris coordinate system to the modern frame IS computed by Horizons. (...) The circumstances pertaining to the regeneration of the spacecraft trajectory source files are not well known.’

General remarks on the limitations of the accuracy of JPL ephemerides are found in chap. 19 of the HORIZONS manual [10].

2.2 General method

We used the computer algebra system *Mathematica* to calculate spacecraft trajectories from known initial conditions and from the positions of gravitating planetary bodies. The built-in-function `NDSolve` with an explicit Runge-Kutta method was used to integrate the equations of motion

$$\frac{d^2}{dt^2}\vec{r}(t) = -G \sum_i \frac{m_i(\vec{r} - \vec{r}_i)}{|\vec{r} - \vec{r}_i|^3}; \quad \vec{r}(0) = \vec{r}_0; \quad \frac{d}{dt}\vec{r}(0) = \vec{v}_0, \quad (1)$$

while the sum is taken over all relevant bodies. Instead of calculating gravitational forces using the gravitational constant G and masses, the much more accurate Keplerian constants [10], p. 47, for the sun and respective planets were implemented. For simplicity, Mercury, Venus, Earth, Mars and the asteroid belt masses were assumed to stay at sun’s barycenter. Thus, for our approximate analysis ephemerides of the outer planets and the sun were sufficient. Since the same $1/r^2$ -law is obeyed, radiation pressure was implemented by slightly diminishing the sun’s effective mass (see code below for details)¹. We estimated an effective surface² of the spacecraft of $5.9m^2$ and an albedo of 0.7.

2.3 Data acquisition

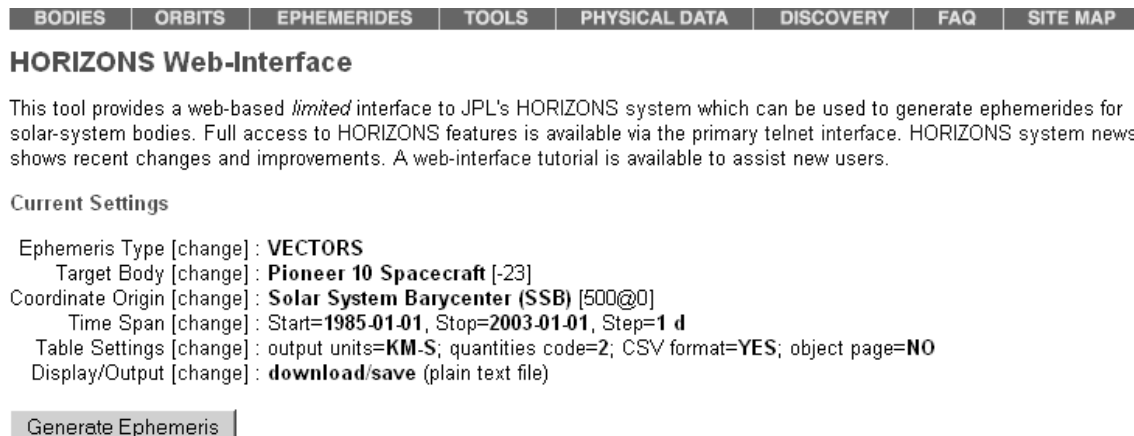


Figure 1: Screenshot of HORIZONS site

¹For large distances, the antenna can be assumed to be directed to the sun.

²Calculated from the diameter of the antenna as given on p. 2 [2]; The value given on p. 28 is different.

All the data needed were downloaded from JPL’s ephemeris site HORIZONS which contains spacecraft trajectories and planetary orbits. With our code, the reader can investigate any period of interest. For our analysis, we downloaded the orbits of the sun, of Jupiter, Saturn, Uranus and Neptune³ and the trajectories of Pioneer 10 and 11. We chose cartesian coordinates (setting VECTORS) with the solar system barycenter as origin. A time step of 1 day was sufficient for our purposes⁴, though the code works with smaller steps without changes. For the numerical treatment, data were 3-D-spline interpolated. Due to lack of information, we did not do any maneuver modeling.

2.4 Modeling

The beginning of our analysis in 1987 coincides with the analysis carried out by [2], see also [8]. The last signal from Pioneer 10 was received in 2002, while the RTG of Pioneer 11 became inoperable in Sept 1995. No reasonable analysis can be done after those dates; we do not even know if the very last signals have been included for the generation of the trajectory files. The time spans for the downloaded files can be chosen from 1985-2003 anyway, since start and end can be chosen separately in *simulatespacecraft*[]. No spacecraft data is available before the respective starts in 1972 and 1973.

3 Results - a preliminary analysis

We compare predicted (simulated with conventional gravity) and observed (HORIZONS) values for position, velocity, and acceleration (fig. 2 and 3). All quantities show a significant deviation.

3.1 Comparison with position

Here we consider the predicted $r(t)$ of our simulation with the position $r_H(t)$ in the ephemeris file⁵. Velocities and accelerations are obtained by numerical differentiation. Though Pioneer 10 and 11 do not move precisely in the direction away from the solar system barycenter, for simplicity the radial components only are analyzed.

An anomalous acceleration is clearly visible for both Pioneer 10 and 11 (bottom graphs). While the median (minimizes absolute deviation) is $-7.60 \times 10^{-10} ms^{-2}$ and $-8.29 \times 10^{-10} ms^{-2}$ respectively, a quadratic fit to the acceleration data yields $-7.25 \times 10^{-10} ms^{-2}$ and $-7.05 \times 10^{-10} ms^{-2}$ (Pioneer 10 and 11).

The anomalous velocity decrease is shown in the middle of fig. (2) and (3). A simple estimate $a = \frac{\Delta v}{\Delta t}$ from the velocity data yields $-10.34 \times 10^{-10} ms^{-2}$ for Pioneer 10 and $-8.72 \times 10^{-10} ms^{-2}$ for Pioneer 11.

The deviation in radial position is shown on top of fig. (2) and (3). In this case, the estimate $a = \frac{2\Delta r}{(\Delta t)^2}$ yields $-13.7 \times 10^{-10} ms^{-2}$ and $-9.52 \times 10^{-10} ms^{-2}$.

Of course, different time spans yield slightly different values, as the reader may easily verify. All the above values are calculated automatically by our program.

³Other bodies may easily be included; download the respective ephemeris, add the name to variable planetnames, and set the variable plmax to the desired value.

⁴Flyby modeling would require much smaller steps and quadrupole moments of planets.

⁵While running *simulatespacecraft*[...], set option *vflag* to 0.

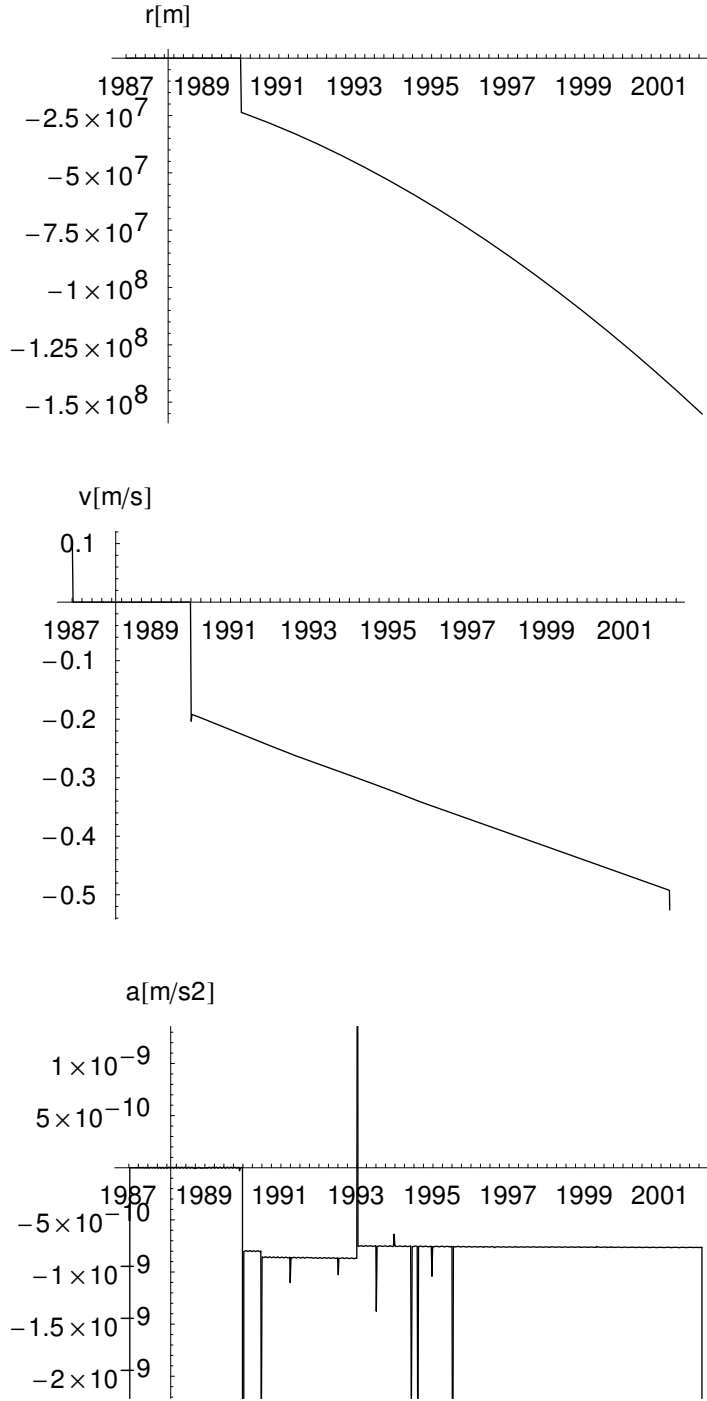


Figure 2: Anomalous deviation (r), velocity (v) and acceleration (a) of Pioneer10 from 1987-2002.

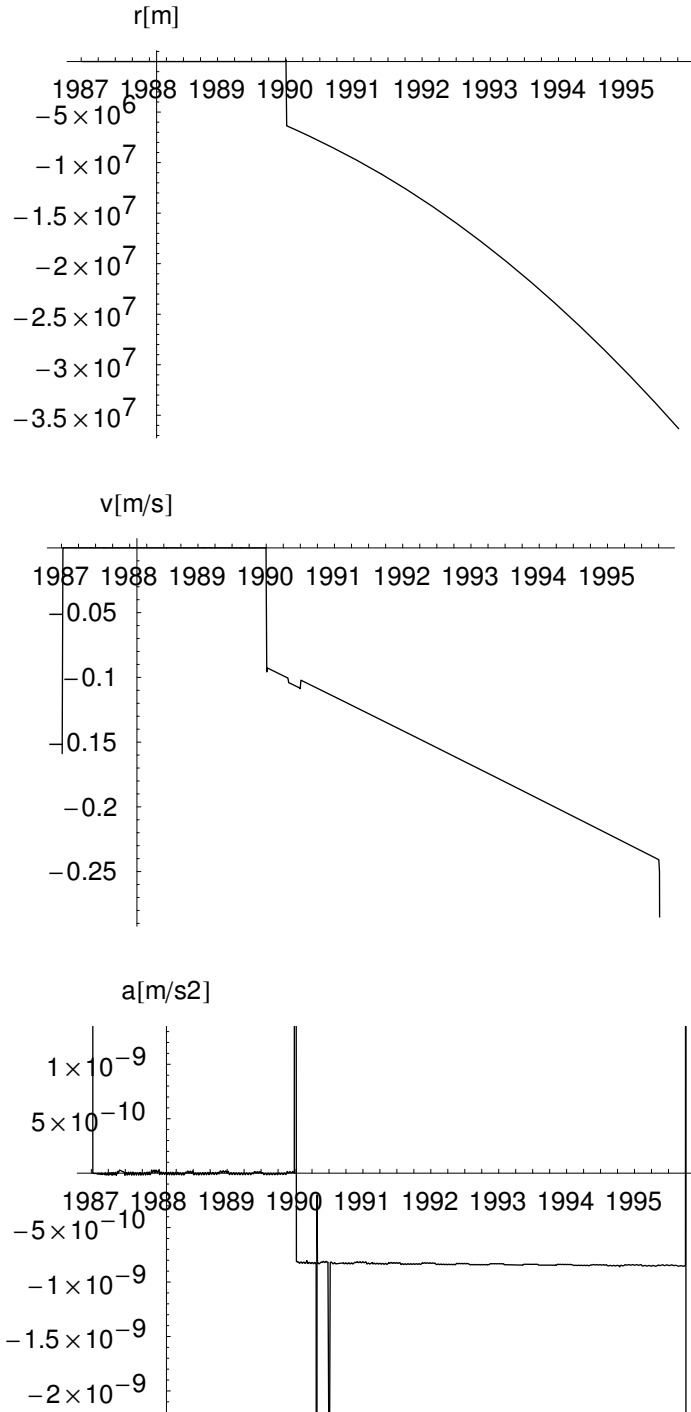


Figure 3: Anomalous deviation (r), velocity (v) and acceleration (a) of Pioneer11 from 1987-1995.

3.2 Comparison with velocities

The derivatives $v(r)$ of our predicted $r(t)$ from the simulation are here compared to the velocities $v_H(t)$ from HORIZONS⁶. Calculating a from Δv in this case yields $-13.2 \times 10^{-10} ms^{-2}$ for Pioneer 10 and $-4.56 \times 10^{-10} ms^{-2}$ for Pioneer 11. The anomalous acceleration functions (see bottom graphs of fig. 2 and 3) do practically not change. Run *simulatespacecraft*[-1,...,1] and *generateplots*[-1] to get the respective plot.

4 Discussion

Though the deviation of the observed quantities from the predicted ones are clearly visible and confirm the order of magnitude of $a_p \approx cH_0$, there are a couple of results we cannot understand yet.

First of all, there is a big jump in the data on January 1st, 1990. Before that date, we cannot verify the known anomalous acceleration at all, there seems to be an agreement with the predicted trajectories. This must be due to a systematic error in the ephemeris programs, i.e. a mismatch between data used then and now, as mentioned above in the header of the JPL ephemeris file.

Further jumps in the acceleration of Pioneer 10, though of much smaller amount, occurred on January 1st, 1993 and in June 1990. The acceleration remains in the range of a_p , however. While these jumps occur from a more or less constant level to another, there are a couple of isolated disturbances, see bottom of fig. (2) and (3). Most likely those disturbances are due to spacecraft maneuvers we did not model at all. Taking the median, those disturbances are practically taken out from the analysis. One should keep in mind however that even the estimates from Δr and Δv (which were affected by maneuvers) yielded a_p in the correct order of magnitude. Thus in no case the Pioneer anomaly can be an artifact of maneuver mismodeling.

Looking at the acceleration plots at a high resolution we did not show here, a sinewave disturbance with amplitude of about $10^{-11} ms^{-2}$ appears. The period however varies continuously from about 12 days (Pioneer 11, around 1987) to 55 days (Pioneer 10, around 1997) and therefore cannot be attributed to a mismodeling of lunar ephemerides or a neglect of tidal effects of the receiver station. It is clearly *not* a manifestation of the annual and diurnal signal published in [2] and must be due to some other systematic error.

5 Conclusions

Despite the limitations of JPL's ephemeris data, we could verify the anomalous acceleration of Pioneer 10 and 11, i.e. a_p cannot be due to a mismodeling of gravitational attraction, maneuvers or radiation pressure. With our code, the reader can easily verify or falsify further hypotheses on the origin of the anomaly. To account for the effect of dust, Kuiper or asteroid belt masses, dark energy etc. it suffices to run the program with different parameters or to add minor changes to the code. We hope that this hands-on demonstration will develop further the interest and critical acceptance of this outstanding effect.

Acknowledgement. Though we are grateful for any comments, please understand that we cannot guarantee functionality or give further support for getting this program to run on your computer.

References

- [1] J. D. Anderson, P. A. Laing, E. L. Lau, A. S. Liu, M. M. Nieto, and S. G. Turyshev. Indication, from Pioneer 10/11, Galileo, and Ulysses Data, of an Apparent Anomalous, Weak, Long-Range Acceleration. *Physical Review Letters*, 81:2858–2861, October 1998.

⁶While running *simulatespacecraft*[:,], set option *vflag* to 1.

- [2] J. D. Anderson, P. A. Laing, E. L. Lau, A. S. Liu, M. M. Nieto, and S. G. Turyshev. Study of the anomalous acceleration of Pioneer 10 and 11. *Physical Review D*, 65(8):082004, April 2002, arXiv: gr-qc/0104064.
- [3] M. M. Nieto and S. G. Turyshev. Finding the origin of the Pioneer anomaly. *arXiv*, gr-qc/030817, 2003.
- [4] M. M. Nieto and S. G. Turyshev. The Pioneer anomaly: The data, its meaning, and a future test. *arXiv*, gr-qc/0411077, 2004.
- [5] V.G. Toth and S. G. Turyshev. The Pioneer anomaly: seeking an explanation in newly recovered data. *arXiv*, gr-qc/0603016, 2006.
- [6] M. M. Nieto. The Quest to Understand the Pioneer Anomaly. *ArXiv: gr-qc/0702017*, February 2007.
- [7] J.D. Anderson, J.K. Campbell, and M.M. Nieto. The energy transfer process in planetary flybys. *ArXiv*, astro-ph/0608087, 2006.
- [8] C. Lämmerzahl, O. Preuss, and H. Dittus. Is the physics within the solar system really understood ? *ArXiv*, gr-qc/0604052, 2006.
- [9] A. Unzicker. Why do we Still Believe in Newton’s Law ? Facts, Myths and Methods in Gravitational Physics. *ArXiv: gr-qc/0702009*, February 2007.
- [10] JPL. Horizons manual. <ftp://ssd.jpl.nasa.gov/pub/ssd/Horizons.doc.pdf>, 2005.

6 Appendix: data preparation and source code

6.1 Step-by step procedure in 15 minutes

1. Create your directory ‘pioneer’ and copy all of the following files in there
2. Goto NASA’s ephemeris site HORIZONS: <http://ssd.jpl.nasa.gov/horizons.cgi>
3. Change ephemeris type to VECTORS (vector table)
4. Set Coordinate origin to [500@0], ecliptic and mean equinox of reference epoch ICRF/J.2000.0
5. Set time span from 1985-01-01 to 2003-01-01, step 1 day
6. Chose table settings *km, km/s*, quantities code = 2 (two-state vector x, y, z, vx, vy, vz), CSV format = *YES*, object page = *NO*.
7. Display/Output: download/save as plain text file.
8. Chose target body: sun (sol)
9. Generate ephemeris and save as *sun-85-03.txt*. See also screenshot fig. 1.
10. Proceed in the same manner with Jupiter, Saturn, Uranus, Neptune, Pioneer 10 and 11 as target bodies.
11. Cross-check if the file ending for the Pioneers corresponds to the last two entries of ‘spans’ (line 2 of the code).

12. Chose ‘barycenter’ where appropriate (otherwise you’ll miss Jupiter’s satellites) and save as *neptune-85-03.txt* etc.
13. Type source from appendix or download it from *www.alexander-unzicker.de/pioneer.txt*⁷
14. Change in the first line the path to your pioneer directory
15. Open a Mathematica *.nb file and run the following commands (see also file end of source):

```

SetDirectory["c:\\yourpionerdirectory"]; (* insert a \\ for any \ in the path*)
<<pioneer.txt;
planetini[5,"-85-03"];
simulatespacecraft[-1, 46800.5(*day start 5.1.87*), 8.75(*insert years*), 0 (*option vflag*)];
generateplots[-1];

```
16. Proceed likewise with *simulatespacecraft*[-2, 46798.5(* 3.1.87*), 15.08,0] for Pioneer 10.

6.2 Source code

```

(***** global settings *****)
planetnames = {"sun", (*"merkur", "venus", "earth", "mond", "mars", *)"jupiter", "saturn", "uranus",
"neptune", "pluto", "pioneer10", "pioneer11"};
spans={"-85-03", "-85-03"}; (*fileendings of spacecraft emphemerides*)
mjd=2400000; (** mean julianian day correction**)
day=3600*24; (* SI unit is seconds *)
(***** constants for radiation pressure modeling *****)
cc = 299792458; (*speed of light*) AU = 1.496*10^(11); (* astronomic unit *)
albedo=0.7; solarK=1367*AU^2/cc*(1+albedo);
surfaceP={5.9,5.9}; (* effective surfaces for radiation pressure in m^2 *)
massP={258,259}; (* spacecraft masses*)
(*Instead of masses, the much more accurate Kepler constants are used, see HORIZONS manual p.47 ****)
KKs=10^9{132712440017.98698,126712767.857796, 37940626.061137281,5794549.00707,
6836534.0638792608,981.6008877}; (*kepler's constants**)
(** all masses inside the asteroid belt (3 10^(21) kg ) added to sun (KKs[[1]]) **)
KKs[[1]]+=10^9*Apply[Plus, {22032.080486417923, 324858.59882645978,398600.43289693922,
4902.8005821477636, 42828.314258067119, 200 (*asteroid belt estimate*)}];
(** for plots: display year numbers at axes instead of days *)
subti=4; ti87 =Transpose[{Table[46796.5+365.25i/subti,{i,0,16subti}],
Table[If[IntegerQ[i/subti]==True,1987+i/subti,""],{i,0,16 subti}]}];
ti88 =Transpose[{Table[46796.5+365.25i/subti,{i,0,16 subti}],
Table[If[IntegerQ[i/subti/2]==True,1987+i/subti,""],{i,0,16 subti}]}];
SCplots=Table[{0,0,0}, {i,10}]; (* plot variable*)
$DefaultFont = {"Arial", 8};
readplanet[name_,ending_.]:=Block[{qwe, wer}, filename=planetnames[[name]] < >ending < >".txt";
If[FileInformation[filename]=={},Print["Missing ephemeris file. Stop."];Break[]];
IO=3; (* interpolation order of splines *)
qwe = Import[filename, "CSV"];
cut1 = Position[qwe, "$$SOE"][[1, 1]]; cut2 = Position[qwe, "$$EOE"][[1, 1]];
wer = Take[Drop[qwe, cut1], cut2 - cut1 - 1];
xyz = 1000*Transpose[Take[Transpose[wer], {3, 5}]]; (** km - m factor **)

```

⁷Do not paste and copy from LaTeX source code, this will create error messages.


```

vxyz = 1000*Transpose[Take[Transpose[wer], {6, 8}]];
time = Flatten[Transpose[wer][[1]]] - mjd;
x1=Interpolation[Transpose[{time,Transpose[xyz][[1]]}],InterpolationOrder -> IO];
y1=Interpolation[Transpose[{time,Transpose[xyz][[2]]}],InterpolationOrder -> IO];
z1=Interpolation[Transpose[{time,Transpose[xyz][[3]]}],InterpolationOrder -> IO];
(** x,y,z, as function, interpolated**)
vx1=Interpolation[Transpose[{time,day*Transpose[vxyz][[1]]}],InterpolationOrder -> IO];
vy1=Interpolation[Transpose[{time,day*Transpose[vxyz][[2]]}],InterpolationOrder -> IO];
vz1=Interpolation[Transpose[{time,day*Transpose[vxyz][[3]]}],InterpolationOrder -> IO];
];
(* simple differentiation routine for interpolating functions of different range, step: 1 day*)
numdiff[tab_,st_(*start time in JD*),en_(*end time in JD*)]:=Block[{tab1p,tab1m,tab2m, tab2p},
tab1p = RotateRight[tab, 1]; tab1m = RotateRight[tab, -1];
dtab = Flatten[{tab[[2]]-tab[[1]],Drop[Drop[(tab1m - tab1p)/2, {1}], {-1}],tab[[-1]]-tab[[-2]]}/day];
ttime = Table[i, {i, st, en, 1}];
dtabtime = Transpose[{ttime, dtab}];
dlist=Interpolation[dtabtime];dlist;
planetini[plmax_,plfileending_]:=Block[{},
(***** Reading data of all relevant planets 1 sun 2 jup, 3 sat, 4 Ura, 5 nep *****)
xx=yy=zz={};
Print["Initialize planets..."];
For[k=1,k <=plmax,k++,readplanet[k,plfileending];AppendTo[xx,x1];AppendTo[yy,y1];AppendTo[zz,z1]];
simulatespacecraft[craftflag_,start_,yrs_,vflag_(* set to 1 for comparing observed velocities*)]:=Block[{},
(***** starting values *****)
(** -1 for pio 11, -2, for pio 10, anything else runs automatically*)
fileending=spans[[craftflag]];
sta=start;
(* radiation pressure modelled as missing solar mass *)
KKs[[1]]=-solarK*surfaceP[[craftflag]]/massP[[craftflag]];
end=Ceiling[start+yrs*365.25]+.5;
readplanet[craftflag, fileending];
(**** starting values for simulation ****)
{x0,y0,z0}={x1[t],y1[t],z1[t]} /. t->start;
{vx0,vy0,vz0}={vx1[t],vy1[t],vz1[t]} /. t->start; (* new*)
Print["computing trajectory for ", planetnames[[craftflag]], " ..."];
(***** numeric integration of the equations of motion by Runge-Kutta****)
plmax=Length[zz];
loe = NDSolve[{
x''[t]==Apply[Plus,Table[-day^2 KKs[[i]](x[t]-xx[[i]][t])/((x[t]-xx[[i]][t])^2+
(y[t]-yy[[i]][t])^2+(z[t]-zz[[i]][t])^2)^(3/2), {i, 1, plmax}]],
y''[t]==Apply[Plus, Table[-day^2 KKs[[i]](y[t]-yy[[i]][t])/((x[t]-xx[[i]][t])^2+
(y[t]-yy[[i]][t])^2+(z[t]-zz[[i]][t])^2)^(3/2),{i,1, plmax}]],
z''[t]==Apply[Plus,Table[-day^2 KKs[[i]](z[t]-zz[[i]][t])/((x[t]-xx[[i]][t])^2+
(y[t]-yy[[i]][t])^2+(z[t]-zz[[i]][t])^2)^(3/2), {i, 1, plmax}]],
x[start]==x0,y[start]==y0,z[start]==z0,
x'[start]==vx0,y'[start]==vy0, z'[start]==vz0},
{x[t], y[t],z[t]}, {t, start, end},Method->"ExplicitRungeKutta"];
r[t_]:=Sqrt[loe[[1, 1, 2]]^2 +loe[[1, 2, 2]]^2 +loe[[1, 3, 2]]^2]; (* solution r(t) *)
v[t_]:=D[r[t],t]/day;
a[t_]:=D[v[t],t]/day;

```

```

readplanet[craftflag, fileending]; (* reading spacecraft data again*)
radius[t_]:= Sqrt[x1[t]^2 + y1[t]^2 + z1[t]^2]; (* observed r(t) from Horizons*)
vradial[t_]:= (vx1[t]*x1[t] + vy1[t]*y1[t] + vz1[t]*z1[t])/day/radius[t]; (*radial component*)
vobserv=Interpolation[Transpose[{Table[i,{i,start,end,1}],Table[vradial[t],{t,start,end,1}]}]];
ranom[t_]:=radius[t]-r[t]; (* anomaly of position **)
(**** vflag=1 compares to observed velocity, vflag=0, to position****)
If[vflag==1, vvv=numdiff[Table[r[t], {t, start, end, 1}], start, end];
Clear[vanom];vanom[t_]:=vobserv[t]-vvv[t],vanom=numdiff[Table[ranom[t], {t, start, end, 1}],start,end];
aanom=numdiff[Table[vanom[t], {t, start, end, 1}],start,end];
ap=((vanom[t]/.t ->end)-(vanom[t]/.t ->start))/(end-start)/day; (* ap estimate from velocities *)
ap2=2((ranom[t]/.t ->end)-(ranom[t]/.t ->start))/(end-start)^2/day^2; (* ap from position ****)
Print["anomalous acceleration (from v and r): ", ap, " ", ap2];
atab=Table[aanom[t], {t, sta, end}];
median=Median[atab];
abwei=FindMinimum[Sum[Abs[x - atab[[i]]], {i, Length[atab]}], {x, 0}][[2, 1, 2]];
abwei2=FindMinimum[Sqrt[Sum[Abs[x - atab[[i]]]^2, {i, Length[atab]}], {x, 0}][[2, 1, 2]];
Print["Median, absolute, quadratic deviation minimized: ", median, " ", abwei, " ", abwei2];
KKs[[1]]+=solarK*surfaceP[[craftflag]]/massP[[craftflag]]; (*take out again radiation from solar mass*)
generateplots[body_]:=Block[{},AO=sta+400;(* clean plot: axes origin shifted days in the future *)
tic=If[body==-1,ti87,ti88];
SCplots[[body,1]]={Plot[radius[t]-r[t],{t, sta, end}, Ticks->{tic, Automatic},
AxesOrigin->{AO, 0},AxesLabel->{"", "r"},PlotRange->All]};
SCplots[[body,2]]={Plot[vanom[t],{t, sta, end}, Ticks->{tic, Automatic},
AxesOrigin->{AO, 0},AxesLabel->{"", "v"},PlotRange->All]};
SCplots[[body,3]]={Plot[aanom[t],{t, sta, end}, Ticks->{tic, Automatic},
AxesOrigin->{AO, 0},AxesLabel->{"", "a"}]};

```